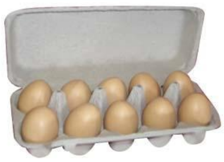
Stage 1

End of Stage Objective:
Children solve problems including halving and sharing
Children solve one step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Children investigate sharing and putting items into groups using items such as egg boxes, ice cube trays and baking tins.

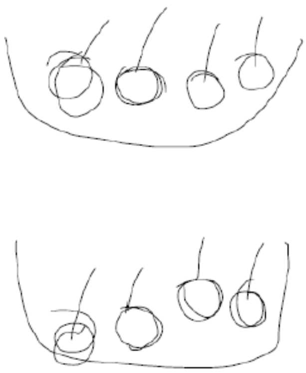


They experience practical calculation opportunities using a wide variety of equipment such as role play, small world play, counters and cubes.

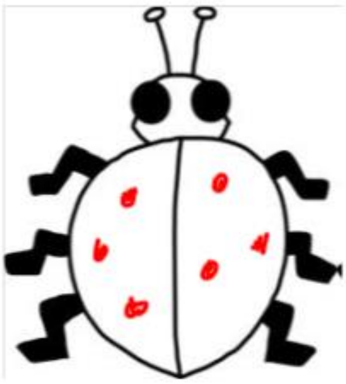
Acting out sharing and talking about whether it is fair.

Sorting objects into groups.

They may develop ways of recording calculations using pictures.



Showing how they shared the apples at snack time between two groups.



Having six spots between two sides of a ladybird.

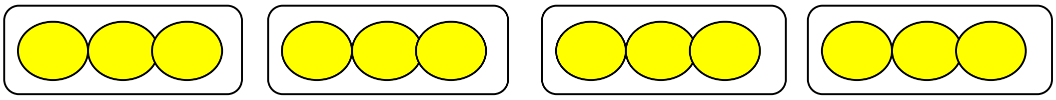
The concept of remainders should be discussed as it arises. 'There is one more', 'Each bowl has four apples and there is one more'.

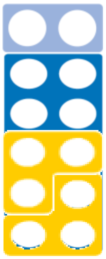
Stage 2

End of Stage Objective:
Calculate mathematical statements for division within the multiplication tables and write them using the division and the equals sign.

Children will use practical equipment to represent division as grouping (repeated addition) and use jottings to support their calculations.

12÷3= (read as how many groups of 3 are there in 12?)

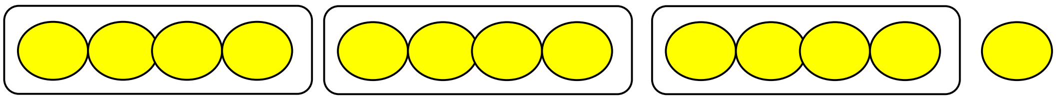




Covering the 12 with shapes of 3 provides clear imagery.

Children need to develop their understanding of division with remainders.

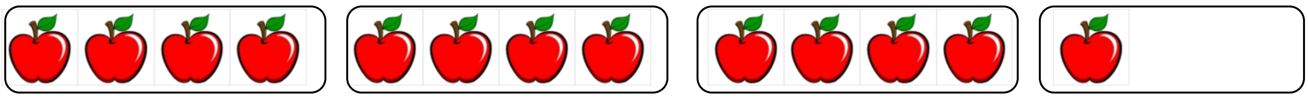
13÷4=



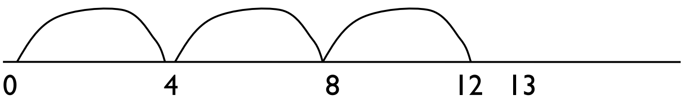
They need to be able to make decisions about what to do with remainders after division and round up or down according to the context.

For example

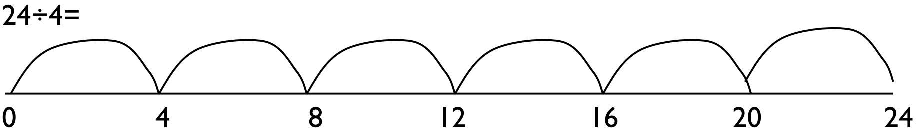
Apples are packed in boxes of 4. There are 13 apples. How many boxes do I need? **4 boxes as all apples need to be in a box.**



As they become secure they can show this on a blank number line making explicit links to their times tables. They need to develop more efficient ways of working with larger numbers.



24÷4=

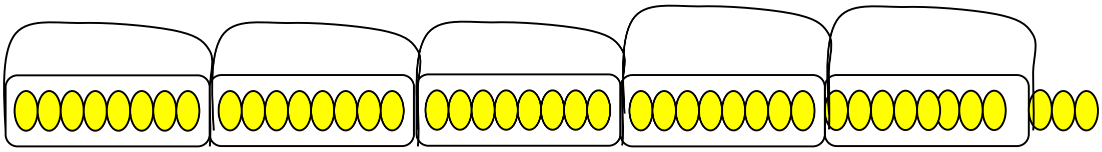


Stage 3

End of Stage Objective:
Write and calculate mathematical statements for division using the multiplication tables that they know, including for two- digit numbers divided by one- digit numbers, progressing to formal methods.

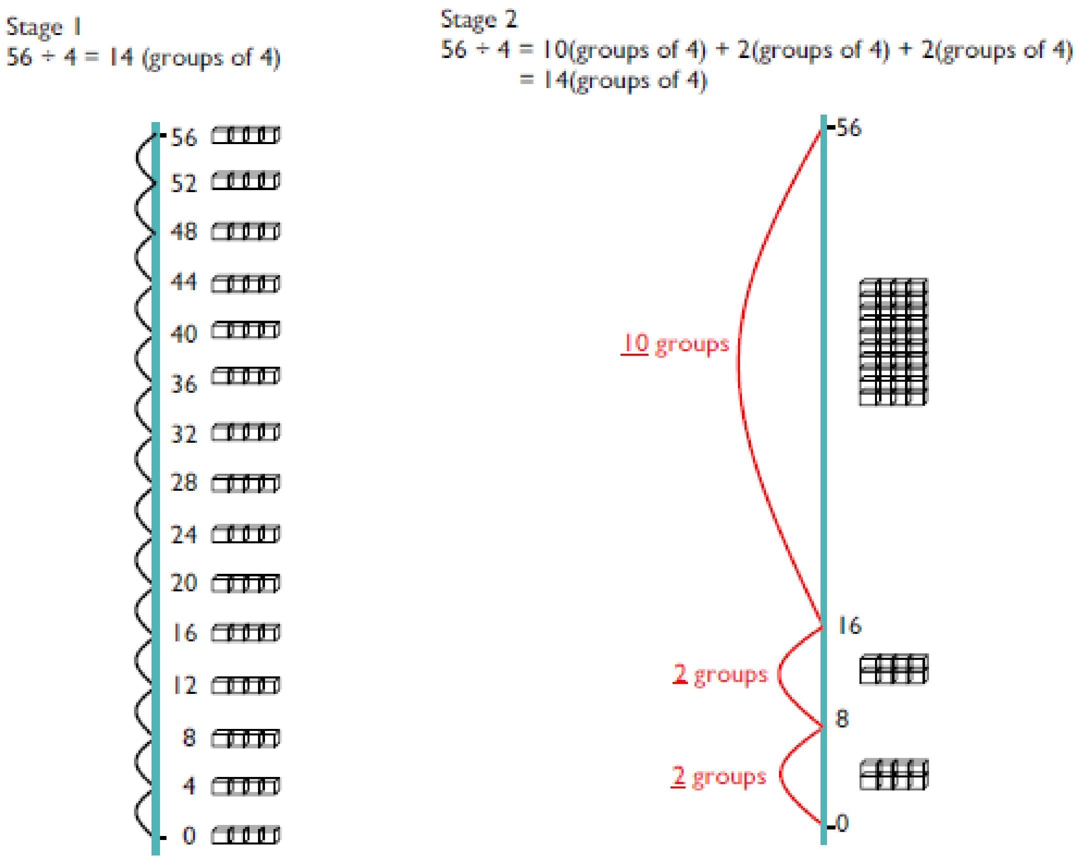
Initially the children will continue to use division as grouping (including those with remainders) where appropriate linked to the multiplication tables that they are familiar with.

43÷8=



43÷8= 5 remainder 3

They should quickly progress to more formal and efficient methods. Relevant equipment should be used consistently to provide imagery for the transition.

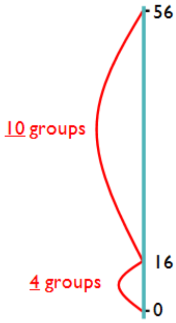


Children should be able to solve real life problems including money and measures. They need to be able to make decisions about what to do with remainders after division and round u
Or down accordingly.

Stage 4

End of Stage Objective:
Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

<u>Children should be secure in column subtraction before moving onto this stage.</u>



The number line method used in stage 3 can be linked to the chunking method to enable children to make links in their understanding.

This image can be used to reinforce the link between a numberline and the expectations of this stage (NOT as a method!).

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. **<u>Initially without remainders.</u>**

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

14
4) 56
- 40 10x
16
- 16 4x
0

Answer: 14

Key facts box

1x	4
2x	8
5x	20
10x	40

73 ÷ 3

Progress to two digit by one digit **<u>including remainders.</u>**

24r1
3) 73
- 30 10x
43
- 30 10x
13
- 6 2x
7
- 6 2x
1

Key facts box

1x	3
2x	6
5x	15
10x	30

196 ÷ 6

By the end of stage 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key box can be extended to include more number facts.

32r4
6) 196
- 120 20x
76
- 60 10x
16
- 12 2x
4

Key facts box

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Stage 5

End of Stage Objective:
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

To start stage 5, the children will continue to use the chunking method initially.

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. Any remainders should be shown as integers, e.g.

523 ÷ 8

```
      65r3
  8 ) 523
    - 320    40x
      203
    - 160    20x
       43
    -  40     5x
        3
```

By the end of stage 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x.

2458 ÷ 7

```
       351r1
  7 ) 2458
    - 2100    300x
       358
    -  350     50x
         8
    -    7      1x
         1
```

Short division

Children who are secure with chunking can be introduced to **short division** (bus stop), initially with TU ÷ U (if the teacher feels they are ready). Chunking will continue to be used in stage 6 as **long division,** so children need to realise they will still use both methods.

Deines and place value counters can be used to support the conceptual understanding when introducing the method. Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

72 ÷ 3 = 24

```
      2 4
  3 ) 7 ¹2
```

Following this, children can progress to HTU ÷ U and THTU ÷ U using the same method. e.g.

196 ÷ 6

```
        3  2  r4
  6 )  1 ¹9 ¹6
```

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly. For example 240 ÷ 52 is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

Children should be confident with both methods, as chunking will be required in stage 6 for long division.

By the end of stage 5, children should be able to use these methods to answer the following:

Short division

98 ÷ 7 becomes

```
      1   4
  7 | 9  ²8
```

Answer: 14

432 ÷ 5 becomes

```
      8   6  r2
  5 | 4  3  ³2
```

Answer: 86 remainder 2

Stage 6

End of Stage Objective:
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Use written division methods in cases where the answer has up to two decimal places.

In stage 6, the children will be developing methods for both short division and long division, including decimals and answers up to two decimal places.

Short division

Children will continue to use written methods to solve short division TU ÷ U and HTU ÷ U.

Any remainders should be shown as fractions, i.e. if the children were dividing 196 by 6, the answer would be 32 remainder 4, so we write it as 4/6 (4 out of 6), which can be simplified to 2/3.

196 ÷ 6

```
      3 2  r 4/6
6 ) 1 ¹9 ¹6
```

4 ← remainder
6 ← divisor

This could be simplified to 32 r 2/3

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

87.5 ÷ 7

```
     1 2 . 5
7 ) 8 ¹7 . ³5
```

Or questions where the answer requires the answer as a decimal up to two places. In the example question, 87÷7=12 r 3, and the remainder of 3 becomes 30 tenths (so 0.7x4=4r2, leaving a remainder of 2 tenths). The two tenths become hundredths, and the children continue to work this way until they have 3 decimal places. You need this in order to round up or down. Therefore the answer to 87 ÷ 7 to two decimal places is 12.43, as we round up the hundredth to 3.

87 ÷ 7

```
     1 2 .  4  2  8
7 ) 8 ¹7 . ³0 ²0 ⁶0
```

Long division

To develop the chunking method further, long division should be extended to include dividing a four-digit number by a two-digit number. Depending on context, children should be able to give answers as remainders or fractions.

432 ÷ 15 becomes

```
          2   8   r 12
1   5 | 4   3   2
        3   0   0
        1   3   2
        1   2   0
            1   2
```

Answer: 28 remainder 12

432 ÷ 15 becomes

```
          2   8
1   5 | 4   3   2
        3   0   0     15×20
        1   3   2
        1   2   0     15×8
            1   2
```

$\frac{12}{15} = \frac{4}{5}$

Answer: 28 $\frac{4}{5}$

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places, using their knowledge of fractions e.g.

3574 ÷ 8

```
8 ) 3574
  - 3200    400x
     374
  -  320     40x
      54
  -   48      6x
       6
```

$\frac{4}{6}$ ← remainder / divisor

So 3574 ÷ 8 is $446\frac{6}{8}$
(when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2}$ = 0.5
Quarters: $\frac{1}{4}$ = 0.25, $\frac{3}{4}$ = 0.75
Fifths: $\frac{1}{5}$ = 0.2, $\frac{2}{5}$ = 0.4, $\frac{3}{5}$ = 0.6, $\frac{4}{5}$ = 0.8
Tenths: $\frac{1}{10}$ = 0.1, $\frac{2}{10}$ = 0.2, $\frac{3}{10}$ = 0.3, $\frac{4}{10}$ = 0.4, $\frac{5}{10}$ = 0.5, $\frac{6}{10}$ = 0.6, $\frac{7}{10}$ = 0.7, $\frac{8}{10}$ = 0.8, $\frac{9}{10}$ = 0.9

and reduce other equivalent fractions to their lowest terms.
In the example above, 3574 ÷ 8, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms. As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 446.75.

Using fractions equivalents is not possible, the children can use their knowledge of multiplication facts to find the two decimals places, e.g.

362 ÷ 17

```
      21.29
17 ) 362
   - 340      20x
      22
   -  17       1x
      5.0
   -  3.4     0.2x
      1.60
   -  1.53    0.09x
      0.07
```

To enable children to continue the calculation, they need to understand that 5 is the same as 5.0

When recalling and deriving multiplication and division facts, children should also identify decimal equivalents of times tables,
e.g. if 2 x 17 = 34, I know that 0.2 x 17 = 3.4
if 9 x 17 = 153, 0.9 x 17 = 15.3
so 0.09 x 17 = 1.53

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

The DIAMOND Learning Partnership Trust

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